

Off-Shell Supersymmetry

Chiu Man Ho^{1,*} and Nobuchika Okada^{2,†}

¹*Department of Physics and Astronomy,
Michigan State University, East Lansing, MI 48824, USA*

²*Department of Physics and Astronomy,
University of Alabama, Tuscaloosa, AL 35487, USA*

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Abstract

Supersymmetry does not dictate the way we should quantize the fields in the supermultiplets, and so we have the freedom to quantize the Standard Model (SM) particles and their superpartners differently. We propose a generalized quantization scheme under which a particle can only appear off-shell, while its contributions to quantum corrections are exactly the same as those in the usual quantum field theory. We apply this quantization scheme solely to the sparticles in the R -parity preserving Minimal Supersymmetric Standard Model (MSSM). Thus sparticles can only appear off-shell. They could be light but would completely escape the direct detection at any experiments such as the LHC. However, our theory still retains the same desirable features of the usual MSSM at the quantum level. For instance, the gauge hierarchy problem is solved and the three MSSM gauge couplings are unified in the usual way. Although direct detection of sparticles is impossible, their existence can be revealed by precise measurements of some observables (such as the running QCD coupling) that may receive quantum corrections from them and have sizable deviations from the SM predictions. Also the experimental constraints from the indirect sparticle search are still applicable.

*Electronic address: cmho@msu.edu

†Electronic address: okadan@ua.edu

I. INTRODUCTION

Supersymmetric (SUSY) extension of the Standard Model (SM) is one of the most promising ways to solve the gauge hierarchy problem in the SM. The Minimal Supersymmetric Standard Model (MSSM) not only offers a solution to the gauge hierarchy problem, but also provides several interesting features for particle physics phenomenology. For example, the three MSSM gauge couplings are beautifully unified at $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, which suggests an interesting paradigm, Grand Unified Theory (GUT). With a conserved R -parity, the lightest sparticle (neutralino) is a primary candidate for the dark matter in the Universe. Besides, electroweak symmetry breaking can be triggered radiatively from a large top Yukawa coupling in the presence of SUSY-breaking terms. The SM-like Higgs boson mass is then predicted as a function of soft SUSY-breaking terms for the third generation squarks.

It has been expected that SUSY is realized most naturally at the TeV scale, and hence sparticles can be discovered in the first run of the Large Hadron Collider (LHC). In spite of a lot of efforts by the LHC, no signal of sparticles has been observed, and the direct sparticle search has been postponed to the LHC Run II with a collider energy 13 – 14 TeV. For a summary of the sparticle search at the LHC, see [1, 2]. The null search results for colored sparticles such as the gluino \tilde{g} and squarks \tilde{q} imply lower bounds on their masses with roughly $m_{\tilde{g}} \gtrsim 1$ TeV and $m_{\tilde{q}} \gtrsim 600$ GeV. The lower bounds on non-colored sparticles such as sleptons, charginos and neutralinos are not so severe so far, with roughly a few hundred GeV.

In principle, it is possible that sparticles might have actually been produced at the LHC but have escaped from the detection because their signals are too difficult to be distinguished from the SM background. This situation occurs when the energy of hadronic/leptonic jets and the missing transverse energy associated with the sparticle cascade decay are significantly reduced in some SUSY models, such as the so-called Compressed SUSY [3, 4], Stealth SUSY [5, 6] and R -parity violating SUSY models [7]. For these models, the lower bounds on sparticle masses can be relaxed up to a few hundred GeV. However, it is not easy to construct such models in a natural way [8, 9]. These models cannot completely hide the direct signal of sparticles forever. With the substantially improved sensitivity at the LHC Run II, it is conceivable that they will be probed and constrained soon.

Contrary to the SUSY models which can tentatively hide the sparticles from direct detec-

tion, we would like to provide an R -parity preserving MSSM scenario that can survive even if sparticles may never be directly observed at the LHC. Our guiding principle is to retain simplicity and naturalness as much as possible. This work is also based on our observation that we do not need on-shell sparticles in order to solve the gauge hierarchy problem and unify the three MSSM gauge couplings.

In this paper, we propose a generalized quantization scheme under which a particle can only appear off-shell. We then apply this scheme to quantize the sparticles in the R -parity preserving MSSM while the SM particles are quantized in the conventional way. Thus sparticles can only appear off-shell.¹ Without introducing any new interactions or exacerbating the naturalness, this evades all the direct detection bounds on sparticles. However, the contributions from the sparticles to quantum corrections in our theory are identical to those in the usual quantum field theory (QFT). Therefore, our MSSM retains the same desirable features in terms of quantum corrections as the usual MSSM. For instance, the gauge hierarchy problem is solved and the three MSSM gauge couplings are unified in the usual manner. The experimental constraints from the indirect sparticle search are still applicable and the same as in the usual MSSM. As a result, even though sparticles can only appear off-shell, SUSY is still broken with stringent bounds on flavor and additional CP violations [11].

In the following sections, we first describe our generalized quantization scheme which leads to off-shell particles, and then we resolve some apparent pathological issues associated with it. We apply this generalized quantization scheme solely to the sparticles in the R -parity preserving MSSM, and so all the experimental bounds from the direct sparticle search disappear. Finally, we discuss the collider phenomenology of off-shell sparticles and possibilities to indirectly detect them.

II. GENERALIZED QUANTIZATION

In the usual QFT, a real scalar field ϕ is quantized in the following way:

$$\phi(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2} \sqrt{2\omega_{\mathbf{p}}}} \left(a(\mathbf{p}) e^{-ipx} + a^\dagger(\mathbf{p}) e^{ipx} \right), \quad (2.1)$$

where the annihilation operator $a(\mathbf{p})$ and the creation operator $a^\dagger(\mathbf{p})$ obey the commutation relation $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta^{(3)}(\mathbf{p} - \mathbf{p}')$. This ensures that the equal-time canonical quantization

¹ After this paper appeared on arXiv, we were informed that this possibility was mentioned in [10].

scheme $[\phi(x), \dot{\phi}(x')] = i\delta^{(3)}(\mathbf{x} - \mathbf{x}')$ is satisfied. The vacuum $|0\rangle$ is chosen such that $a(\mathbf{p})|0\rangle = 0$. As we know, if we compute the propagator for ϕ , we will obtain the Feynman propagator.

Contrary to the conventional wisdom, we would like to introduce a generalized quantization scheme. We retain the commutation relation $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta^{(3)}(\mathbf{p} - \mathbf{p}')$, but propose that

$$a|n\rangle = \text{sign}\left(n - \frac{1}{2}\right) \sqrt{\left|n - \frac{1}{2}\right|} |n-1\rangle, \quad (2.2)$$

$$a^\dagger|n\rangle = \sqrt{\left|n + \frac{1}{2}\right|} |n+1\rangle, \quad (2.3)$$

for any integer n which characterizes the number of energy units carried by the state $|n\rangle$. Notice that for simplicity, we have suppressed the dependence on momentum \mathbf{p} in Eq. (2.2) and Eq. (2.3). (Apparently, this quantization scheme may lead to the disastrous negative energies and probably even worse problems. We will hold on and resolve the apparent pathological issues below Eq. (2.10). In fact, the resolution may become self-evident when we move on.) To elaborate, with the above choice, we have $a|0\rangle = -\frac{1}{\sqrt{2}}|-1\rangle$ and $a^\dagger|0\rangle = \frac{1}{\sqrt{2}}|1\rangle$. This procedure implies that

$$\langle 0|a^\dagger(\mathbf{p})a(\mathbf{p}')|0\rangle = -\frac{1}{2}\delta^{(3)}(\mathbf{p} - \mathbf{p}'), \quad (2.4)$$

$$\langle 0|a(\mathbf{p})a^\dagger(\mathbf{p}')|0\rangle = \frac{1}{2}\delta^{(3)}(\mathbf{p} - \mathbf{p}'). \quad (2.5)$$

Thus, the vacuum expectation value of the Hamiltonian for ϕ without normal ordering, $H_\phi = 1/2 \int d^3\mathbf{p} \omega_{\mathbf{p}} (a^\dagger(\mathbf{p})a(\mathbf{p}) + a(\mathbf{p})a^\dagger(\mathbf{p}))$, is

$$\langle 0|H_\phi|0\rangle = 0. \quad (2.6)$$

In other words, the vacuum state $|0\rangle$ is no longer a state with the lowest energy but is simply a state with no particles and thereby zero energy.

Applying the above quantization procedures, the propagator for ϕ turns out to be an average of the usual Feynman propagator with an $+i\epsilon$ prescription and a similar one with an $-i\epsilon$ prescription:

$$\langle 0|T\{\phi(x)\phi(y)\}|0\rangle = \frac{1}{2}\left(G_+(x-y) + G_-(x-y)\right), \quad (2.7)$$

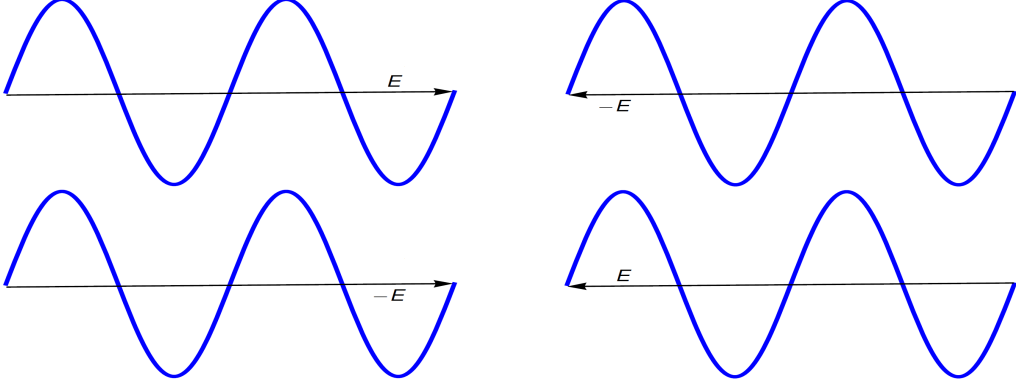


FIG. 1: Propagation of half-retarded and half-advanced energy modes, with the forward time direction pointing to the right. $+E$ and $-E$ energy modes are propagated both forward and backward in time with equal amplitudes.

where $G_+(x-y)$ and $G_-(x-y)$ are respectively given by

$$G_+(x-y) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2 - m^2 + i\epsilon}, \quad (2.8)$$

$$G_-(x-y) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{p^2 - m^2 - i\epsilon}. \quad (2.9)$$

Actually, one can prove that the propagator, $\frac{1}{2} (G_+(x-y) + G_-(x-y))$, is equivalent to an average of the retarded and advanced Green's functions:

$$\begin{aligned} & \frac{1}{p^2 - m^2 + i\epsilon} + \frac{1}{p^2 - m^2 - i\epsilon} \\ &= \frac{1}{2\omega_{\mathbf{p}}} \left(\frac{1}{p_0 - \omega_{\mathbf{p}} + i\epsilon} - \frac{1}{p_0 + \omega_{\mathbf{p}} - i\epsilon} \right) + \frac{1}{2\omega_{\mathbf{p}}} \left(\frac{1}{p_0 - \omega_{\mathbf{p}} - i\epsilon} - \frac{1}{p_0 + \omega_{\mathbf{p}} + i\epsilon} \right) \\ &= \frac{1}{2\omega_{\mathbf{p}}} \left(\frac{1}{p_0 - \omega_{\mathbf{p}} + i\epsilon} - \frac{1}{p_0 + \omega_{\mathbf{p}} + i\epsilon} \right) + \frac{1}{2\omega_{\mathbf{p}}} \left(\frac{1}{p_0 - \omega_{\mathbf{p}} - i\epsilon} - \frac{1}{p_0 + \omega_{\mathbf{p}} - i\epsilon} \right) \\ &= \frac{1}{(p_0 + i\epsilon)^2 - \omega_{\mathbf{p}}^2} + \frac{1}{(p_0 - i\epsilon)^2 - \omega_{\mathbf{p}}^2}. \end{aligned} \quad (2.10)$$

Thus, it is also a half-retarded and half-advanced propagator. Under the generalized quantization scheme, the ϕ particle simultaneously propagates positive and negative energy modes both forward and backward in time with equal amplitudes. Therefore, the ϕ particle behaves like a “standing wave” (in time) — there is *no* net energy flux being transferred. Since an on-shell particle carries a nonzero net energy flux, the ϕ particle propagated by the half-retarded and half-advanced propagator *cannot* be on-shell. The corresponding physical picture is shown in Fig. 1. In other words, the ϕ particle is “ghost-like” in the sense that

it can only appear off-shell. However, the procedure of making the ϕ particle “ghost-like” is completely different from the usual Faddeev-Popov approach [12]. We do not need a ghost Lagrangian. What we need is simply the generalized quantization scheme described above.

Since the ϕ particle can only appear off-shell, the possibility of negative energies does not cause any problem. In fact, one can show that if the state $|n\rangle$ has a negative energy with $n < 0$, it may also acquire a negative norm:

$$\langle n | n \rangle < 0, \quad \text{if } n < 0 \text{ and } |n| = \text{odd}, \quad (2.11)$$

which implies the existence of an indefinite metric.² But again, since the ϕ particle can only appear off-shell, it does not contribute to the unitarity sum of the scattering amplitudes. Hence, unitarity is preserved despite the possible existence of negative-norm states. This is analogous to the Faddeev-Popov ghosts which have negative norms but can only appear off-shell.

Since we have retained $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta^{(3)}(\mathbf{p} - \mathbf{p}')$ in spite of the generalized quantization scheme introduced in Eq. (2.2) and Eq. (2.3), it follows that the equal-time commutator $[\phi(x), \phi(y)]|_{x^0=y^0}$ should vanish as usual. This means that micro-causality is preserved under our framework. Moreover, for spacelike distances with $(x - y)^2 \equiv -r^2 < 0$, there exists a reference frame where $x^0 - y^0 = 0$. It is well-known that

$$G_+(x - y) = \frac{m}{4\pi^2 r} K_1(mr), \quad \text{for } (x - y)^2 < 0, \quad (2.12)$$

where $K_1(z)$ is the modified Bessel function of second kind. Thus outside the light-cone, the usual Feynman propagator predicts that the propagation amplitude is exponentially small but nonzero (as $r \rightarrow \infty$). In contrast, one can show that

$$G_-(x - y) = -\theta(x^0 - y^0) \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}}} e^{-ip(y-x)} - \theta(y^0 - x^0) \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}}} e^{-ip(x-y)}, \quad (2.13)$$

² The idea of indefinite metric was first invented by Dirac [13] and then elaborated by Pauli [14] in 1940s in an attempt to remove the divergences and construct a finite theory of quantum electrodynamics. Their attempt turned out to be not very satisfactory. But the canceling effect due to indefinite metric inspired Lee and Wick to construct an alternative finite theory of quantum electrodynamics [15, 16]. The possible issues of Lee-Wick theory were discussed in [17–19]. For a comprehensive lecture and review on indefinite metric QFT, one can consult [20] and [21] respectively. In any case, due to the success of renormalization, these attempts for taming the divergences were largely forgotten. Recently, the ideas in [15, 16] have been revived to construct a Lee-Wick Standard Model [22–24] which has many interesting properties (including the possibility to solve the gauge hierarchy problem).

and hence we have

$$G_-(x-y) = -G_+(x-y), \quad \text{for spacelike distances with } (x-y)^2 < 0. \quad (2.14)$$

Therefore, our propagator, which is half-retarded and half-advanced, is identically zero for spacelike distances. In other words, the propagator obtained from our generalized quantization scheme predicts that the propagation amplitude is exactly zero outside the light-cone and so it is truly causal.

In fact, our half-retarded and half-advanced propagator may have some resemblance to the absorber theory of radiation proposed by Feynman and Wheeler [25, 26]. They considered a half-retarded and half-advanced electromagnetic field in which electrons radiate symmetrically, both forward and backward in time. As discussed in [27], due to subtle cancellations, the absorber theory is “an apparently acausal theory that is not”. Similarly, under our framework, each of the positive-energy and negative-energy states is propagated symmetrically, both forward and backward in time. The subtle “destructive interference” renders the ϕ particle virtual (off-shell) and, at the same time, causal.

The extension of our generalized quantization scheme to a complex scalar field as well as a vector boson is straightforward. (In the MSSM, we do not have spin-1 sparticles, so the extension to vector bosons is actually irrelevant.) For a complex scalar given by $\Phi(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2} \sqrt{2\omega_{\mathbf{p}}}} (a(\mathbf{p}) e^{-ipx} + b^\dagger(\mathbf{p}) e^{ipx})$, we require the usual commutation relations $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = [b(\mathbf{p}), b^\dagger(\mathbf{p}')] = \delta^{(3)}(\mathbf{p} - \mathbf{p}')$ and impose that similar to $a(\mathbf{p})$ and $a^\dagger(\mathbf{p})$, $b(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ acting on the state $|n\rangle$ satisfy the relations in Eq. (2.2) and Eq. (2.3) respectively. The vacuum expectation value of the Hamiltonian for Φ without normal ordering, $H_\Phi = 1/2 \int d^3 \mathbf{p} \omega_{\mathbf{p}} (a^\dagger(\mathbf{p}) a(\mathbf{p}) + a(\mathbf{p}) a^\dagger(\mathbf{p}) + b^\dagger(\mathbf{p}) b(\mathbf{p}) + b(\mathbf{p}) b^\dagger(\mathbf{p}))$, is $\langle 0 | H_\Phi | 0 \rangle = 0$. Besides, the propagator for Φ , $\langle 0 | T \{ \Phi^\dagger(x) \Phi(y) \} | 0 \rangle$, is of the same form as Eq. (2.7).

We extend our generalized quantization scheme to a Dirac fermion:

$$\psi(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2} \sqrt{2\omega_{\mathbf{p}}}} \sum_{s=\pm} \left(c(\mathbf{p}, s) u(p, s) e^{-ipx} + d^\dagger(\mathbf{p}, s) v(p, s) e^{ipx} \right). \quad (2.15)$$

Here, the creation and annihilation operators obey the usual anticommutation relations $\{c(\mathbf{p}, s), c^\dagger(\mathbf{p}', s')\} = \{d(\mathbf{p}, s), d^\dagger(\mathbf{p}', s')\} = \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{ss'}$. The spinors $u(p, s)$ and $v(p, s)$ satisfy the usual orthogonality conditions as well as the completeness relations: $\sum_s u(p, s) \bar{u}(p, s) = \not{p} + m$ and $\sum_s v(p, s) \bar{v}(p, s) = \not{p} - m$. We introduce the following

quantization steps:

$$c \text{ or } d |n\rangle = \sqrt{\left|n - \frac{1}{2}\right|} |n-1\rangle, \quad (2.16)$$

$$c^\dagger \text{ or } d^\dagger |n\rangle = \sqrt{\left|n + \frac{1}{2}\right|} |n+1\rangle, \quad (2.17)$$

where for simplicity, we have suppressed the dependence on momentum \mathbf{p} and spin index s in Eq. (2.16) and Eq. (2.17). These imply that

$$\langle 0 | c^\dagger(\mathbf{p}, s) c(\mathbf{p}', s') | 0 \rangle = \langle 0 | c(\mathbf{p}, s) c^\dagger(\mathbf{p}', s') | 0 \rangle = \frac{1}{2} \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{ss'}, \quad (2.18)$$

$$\langle 0 | d^\dagger(\mathbf{p}, s) d(\mathbf{p}', s') | 0 \rangle = \langle 0 | d(\mathbf{p}, s) d^\dagger(\mathbf{p}', s') | 0 \rangle = \frac{1}{2} \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{ss'}. \quad (2.19)$$

Notice that due to the anticommutation relations, a difference from the bosonic case is that only $n = -1, 0, 1$ are allowed. One can verify that the norm for each of these states is positive-definite.³ The vacuum expectation value of the Hamiltonian for ψ without normal ordering, $H_\psi = \sum_{s=\pm 1/2} \int d^3 \mathbf{p} \omega_{\mathbf{p}} (c^\dagger(\mathbf{p}, s) c(\mathbf{p}, s) - d(\mathbf{p}, s) d^\dagger(\mathbf{p}, s))$, is

$$\langle 0 | H_\psi | 0 \rangle = 0. \quad (2.20)$$

Similar to the scalars, this means that the vacuum state $|0\rangle$ is no longer a state with the lowest energy but is simply a state with no particles and thereby zero energy.

The propagator for ψ is also half-retarded and half-advanced, where (with spinor indices suppressed) the corresponding propagator $D_-(x-y)$ with an $-i\epsilon$ prescription is given by

$$D_-(x-y) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} \frac{i(\not{p} + m)}{p^2 - m^2 - i\epsilon} \quad (2.21)$$

$$= \theta(x^0 - y^0) \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\not{p} - m}{2\omega_{\mathbf{p}}} e^{-ip(y-x)} - \theta(y^0 - x^0) \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\not{p} + m}{2\omega_{\mathbf{p}}} e^{-ip(x-y)}. \quad (2.22)$$

³ To some extent, $|-1\rangle$ for fermions may be interpreted as a hole state. In contrast, for bosons with $n < 0$ and $|n| = \text{odd}$, $|n\rangle$ acquires a negative-norm. It is then unclear whether this interpretation is valid for bosons. Besides, one may wonder about the corresponding superpartners for the negative-energy states. We note that SUSY is realized at the Lagrangian level, namely the interactions between quantum fields. In the last paragraph of Section III, we argue that SUSY is fundamentally broken in the basis states underlying the generalized quantization scheme. Thus it is not mandatory to worry about the superpartners for the negative-energy states.

The exact form of the propagator for ψ is

$$\begin{aligned} \langle 0 | T \{ \psi_\alpha(x) \bar{\psi}_\beta(y) \} | 0 \rangle &\equiv S(x-y)_{\alpha\beta} \\ &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} (\not{p} + m)_{\alpha\beta} \frac{1}{2} \left(\frac{i}{p^2 - m^2 + i\epsilon} + \frac{i}{p^2 - m^2 - i\epsilon} \right), \end{aligned} \quad (2.23)$$

and so the ψ particle can only appear off-shell. Following the similar arguments for the scalars, the possibility of negative energies is not a problem and micro-causality is preserved. Besides, one can verify, using Eq. (2.22), that this propagator is exactly zero outside the light-cone.

For a Majorana fermion with $\psi = \psi^c \equiv C \bar{\psi}^T$ where C is the charge conjugation matrix, the quantization steps are similar except that we set $c(\mathbf{p}, s) = d(\mathbf{p}, s)$. There are two additional propagators:

$$\langle 0 | T \{ \psi_\alpha(x) \psi_\beta(y) \} | 0 \rangle = [C^{-1} S(x-y)]_{\alpha\beta} \quad (2.24)$$

$$\langle 0 | T \{ \bar{\psi}_\alpha(x) \bar{\psi}_\beta(y) \} | 0 \rangle = [-S(x-y) C]_{\alpha\beta}. \quad (2.25)$$

As we know, it is the integration contour that determines the amount of quantum corrections due to the propagators. In the usual QFT, the propagators behave like $\frac{1}{p^2 - m^2 + i\epsilon}$ whose poles are in the II & IV quadrants. One can close the contour of integration either in the upper or lower half-plane. With the generalized quantization, the propagators for particles behave like an average of $\frac{1}{p^2 - m^2 + i\epsilon}$ and $\frac{1}{p^2 - m^2 - i\epsilon}$. Note that the propagator $\frac{1}{p^2 - m^2 - i\epsilon}$ has poles in the I & III quadrants. Thus, a particle with generalized quantization, *as a single particle*, has poles in all of the four quadrants.

Since particles with generalized quantization have unusual pole structures, it is conceivable that unconventional integration contours are needed for them. We prescribe the integration contours for particles with generalized quantization in Fig. 2, where the x -axis and y -axis represent $\text{Re } p_0$ and $\text{Im } p_0$ respectively. For the left contour, only the possible poles in the III & IV quadrants are picked up. For the right contour, only the possible poles in the I & II quadrants are picked up. Either of the left or right contour allows the poles of $\frac{1}{p^2 - m^2 + i\epsilon}$ and $\frac{1}{p^2 - m^2 - i\epsilon}$ to contribute constructively. With either contour in Fig. 2, one could verify that quantum corrections from particles with generalized quantization are exactly the same as those in the usual QFT.

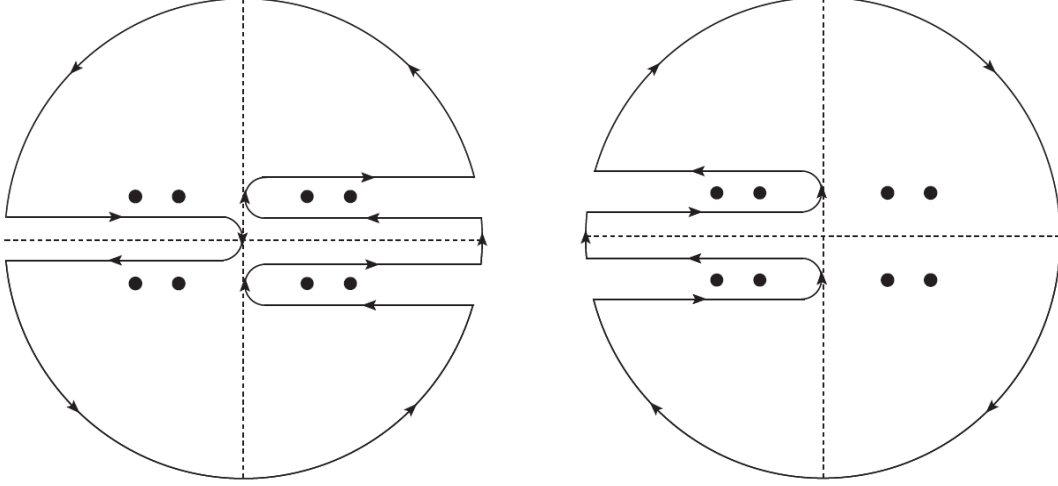


FIG. 2: The integration contours prescribed for particles with generalized quantization. The x -axis and y -axis represent $\text{Re } p_0$ and $\text{Im } p_0$ respectively. The symbols \bullet denote the possible poles (not drawn to scale). For the left contour, only the possible poles in the III & IV quadrants are picked up. For the right contour, only the possible poles in the I & II quadrants are picked up.

III. SUSY WITH GENERALIZED QUANTIZATION

For a given supermultiplet, supersymmetry exhibited at the Lagrangian does not dictate the way we should quantize the fields. Providing the usual R -parity preserving MSSM Lagrangian, we propose that the SM particles are quantized in the conventional way, while their superpartners are quantized according to the generalized scheme described above.⁴ Therefore, sparticles can only appear off-shell in our theory, which evades the direct detection at any experiments. However, since quantum corrections from the sparticles are identical to those in the usual QFT, the gauge hierarchy problem is solved in the usual way.

With the soft SUSY-breaking terms, our MSSM scenario with sparticles obeying the generalized quantization leads to the same attractive phenomenological consequences as the usual MSSM does [28]. For example, the three MSSM gauge couplings are unified at $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, the electroweak symmetry is radiatively broken, and the SM-like

⁴ It would be appealing if one could find a theoretical motivation for making this choice of quantization. A more fundamental theory would probably exhibit the property that the Hilbert spaces for the SM particles and their superpartners are different. However, this is beyond the scope of the current paper which concerns more about the phenomenology. We are actively working on this issue and hope to report elsewhere soon.

Higgs boson mass is obtained through stop loop corrections. Although all the experimental constraints from the direct detection disappear, the results from the indirect sparticle search are still applicable to our MSSM scenario to constrain the sparticle mass spectrum. Similar to the usual MSSM, this requires SUSY to be broken with stringent bounds on flavor and additional CP violations.

As a consequence of applying the generalized quantization scheme to sparticles, we obtain $\langle 0 | H_{\text{sparticles}} | 0 \rangle = 0$. This implies that $\langle 0 | H_{\text{SM}} + H_{\text{sparticles}} | 0 \rangle \neq 0$ even if SUSY is manifest. Thus, it appears that SUSY is fundamentally broken in the basis underlying the generalized quantization scheme. However, the off-shell MSSM Lagrangian is exactly the same as the original MSSM Lagrangian including the soft terms. The structure of quantum corrections (in a typical SUSY theory) responsible for the potential cancellations of divergences is still retained, so the SUSY breaking in the vacuum state has no practical effect on particle physics phenomenology. We can simply remove the constant vacuum energy by normal ordering, as in the usual QFT.

IV. PHENOMENOLOGY OF OFF-SHELL SUSY

As mentioned above, although it is impossible to directly detect sparticles which can only appear off-shell, their existence can be indirectly identified through their contributions to quantum corrections for some observables. For example, the fine structure constant at the Z-pole (m_Z) is very precisely measured as $\alpha_{\text{em}}(m_Z)^{-1} = 127.918 \pm 0.019$ [29], which is consistent with the SM prediction for the evolution of the fine structure constant from low energy to the Z-pole. If charged sparticles such as squarks, sleptons and charginos are involved in the evolution, the resultant fine structure constant at the Z-pole will be altered from the SM prediction. This sets the lower bound on the charged sparticle masses as $\tilde{m} \gtrsim m_Z$.

The discussion about the fine structure constant may give us an idea about how to identify the existence of light colored (off-shell) sparticles such as the gluino and squarks at the LHC, even though the experimental data would show no indication of sparticle productions. At energies higher than their masses, the colored sparticles are involved in the running QCD coupling and will deflect the running from the trajectory predicted by the SM. Employing the 1-loop renormalization group equation for the QCD coupling, we define a deviation of

the running QCD coupling at an energy scale μ as

$$\Delta(\mu) \equiv \frac{\alpha_s^{\text{MSSM}}(\mu)}{\alpha_s^{\text{SM}}(\mu)} - 1 \quad (4.1)$$

$$\approx \frac{\alpha_s^{\text{SM}}(\tilde{m})}{2\pi} (b_{\text{MSSM}} - b_{\text{SM}}) \ln\left(\frac{\mu}{\tilde{m}}\right), \quad (4.2)$$

where α_s^{SM} is the running SM QCD coupling, α_s^{MSSM} is the running QCD coupling with the contributions from colored particles with a degenerate mass \tilde{m} , and $b_{\text{SM}} = -7$ (b_{MSSM}) is the QCD beta function coefficient in the SM (MSSM). Using $\alpha_s(M_t) = 0.0928$ with a top quark pole mass $M_t = 173.34$ GeV [30], we find $\Delta(1 \text{ TeV}) \approx 3.6\%$, for $\tilde{m} = 500$ GeV and $b_{\text{MSSM}} = -3$ due to the contributions from the degenerate gluino and three generations of squarks. This deviation is slightly smaller than the error of the current measurement of the QCD coupling constant in the TeV range [31, 32]. However, we obtain $\Delta(\mu \gtrsim 3 \text{ TeV}) \gtrsim 9\%$ for the same parameters. Therefore, a more precise measurement of the QCD coupling at sufficiently higher energy and luminosity may reveal the off-shell colored particles.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we propose a novel MSSM scenario where particles are quantized under a generalized scheme to only appear off-shell. However, quantum corrections from the sparticles in this theory are exactly the same as those in the usual QFT. As a consequence, most of the phenomenologically attractive properties of the usual MSSM, such as the solution to the gauge hierarchy problem and the successful gauge coupling unification, remain the same. Although direct detection of sparticles is impossible, their existence can be revealed through precise measurements of observables to which off-shell particles give sizable quantum corrections.

In principle, one could apply our generalized quantization scheme to any other theories so as to evade the corresponding bounds from direct detection. Nevertheless, sparticles are particularly well-suited for the generalized quantization. The reason is that the most important merit of sparticles is due to their off-shell quantum contributions. For instance, we do not need on-shell particles in order to solve the gauge hierarchy problem and unify the three MSSM gauge couplings.

Of course, the lightest particle (e.g. neutralino) would no longer be a viable dark matter candidate if it can only appear off-shell. However, this should not be considered as a serious

deficiency of our MSSM scenario. Providing a viable dark matter candidate is a just bonus of the usual MSSM, and it is easy enough to construct other models that give a promising dark matter candidate. Supersymmetry is most crucial for solving the gauge hierarchy problem and unification of the three MSSM gauge couplings.

Our encouraging message is that even if none of sparticles is observed at the LHC Run II, there is still a hope that the sparticles are light but can only appear off-shell. In that case, their existence may have to be indirectly identified through a precise measurement of the running QCD coupling.

In fact, our work is more general than supersymmetry. Our idea has provided a new vision that some new physics may only appear off-shell. An intriguing collider signature of this kind of new physics is that we may see sizable deviations from the SM predictions at precision measurements despite the absence of new particles at direct detection.

Finally, in the present work, the realization of the idea of off-shell supersymmetry relies on the generalized quantization scheme. In order to better justify the consistency of this idea, we have demonstrated that it could be formulated from the path-integral approach in [33].

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- [1] **ATLAS** Collaboration, <https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults>
- [2] **CMS** Collaboration, <https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS>

- [3] T. J. LeCompte and S. P. Martin, Phys. Rev. D **84**, 015004 (2011).
- [4] T. J. LeCompte and S. P. Martin, Phys. Rev. D **85**, 035023 (2012).
- [5] J. Fan, M. Reece and J. T. Ruderman, JHEP **1111**, 012 (2011).
- [6] J. Fan, M. Reece and J. T. Ruderman, JHEP **1207**, 196 (2012).
- [7] R. Barbier, C. Berat, M. Besancon, M. Chemtob, A. Deandrea, E. Dudas, P. Fayet and S. Lavignac *et. al.*, Phys. Rept. **420**, 1 (2005).
- [8] J. L. Feng, Ann. Rev. Nucl. Part. Sci. **63**, 351 (2013).
- [9] N. Craig, arXiv:1309.0528 [hep-ph].
- [10] D. M. Ghilencea, Nucl. Phys. B **876**, 16 (2013).
- [11] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477**, 321 (1996).
- [12] L. D. Faddeev and V. N. Popov, Phys. Lett. B **25**, 29 (1967).
- [13] P. A. M. Dirac, Proc. Roy. Soc. A **180**, 1 (1942).
- [14] W. Pauli, Rev. Mod. Phys. **15** 175 (1943).
- [15] T. D. Lee and G. C. Wick, Nucl. Phys. B **9**, 209 (1969).
- [16] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).
- [17] R. E. Cutkosky, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, Nucl. Phys. B **12**, 281 (1969).
- [18] N. Nakanishi, Phys. Rev. D **3**, 811 (1971).
- [19] D. G. Boulware and D. J. Gross, Nucl. Phys. B **233**, 1 (1984).
- [20] S. Coleman, “Field Theories with Indefinite Metric,” In *Syracuse 1969, Proceedings, Eighth Annual Eastern Theoretical Physics Conference*, Syracuse 1970, 197-216.
- [21] N. Nakanishi, Prog. Theor. Phys. Suppl. **51**, 1 (1972).
- [22] B. Grinstein, D. O’Connell and M. B. Wise, Phys. Rev. D **77**, 025012 (2008).
- [23] J. R. Espinosa, B. Grinstein, D. O’Connell and M. B. Wise, Phys. Rev. D **77**, 085002 (2008).
- [24] B. Grinstein, D. O’Connell and M. B. Wise, Phys. Rev. D **77**, 065010 (2008).
- [25] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945).
- [26] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **21**, 425 (1949).
- [27] S. Coleman, “Acausality,” In *Erice 1969, Ettore Majorana School On Subnuclear Phenomena*, New York 1970, 282-327.
- [28] For a review, see, for example, S. P. Martin, Adv. Ser. Direct. High Energy Phys. **21**, 1 (2010).
- [29] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

- [30] See, for example, D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, JHEP **1312**, 089 (2013).
- [31] S. Chatrchyan *et al.* [CMS Collaboration], Eur. Phys. J. C **73**, 2604 (2013).
- [32] V. Khachatryan *et al.* [CMS Collaboration], Eur. Phys. J. C **75**, 186 (2015).
- [33] C. M. Ho, arXiv:1506.00319 [hep-ph].